The Design of High Speed Low Power Digital FIR Filters Based on Frequency-Response Masking Technique

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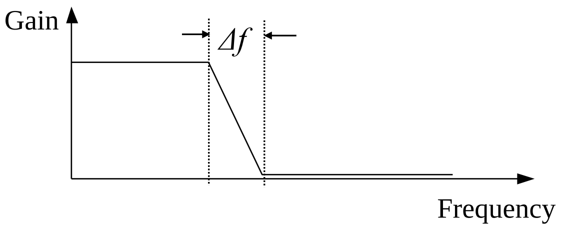
Outline

- A brief introduction to digital filters
- How to achieve high-speed with less power
- The frequency-response masking technique
- Conclusion

Digital Filters

- What is the digital filter?



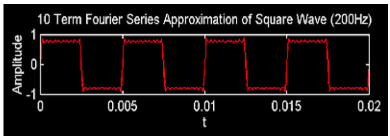
- Two types of filters – Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters.

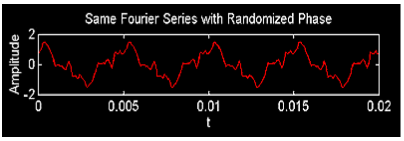
Advantages and Disadvantages

- Advantages :
  - FIR: exact linear-phase characteristic, intrinsically stable implementation.
  - IIR: computationally efficient.
- Disadvantages :
  - FIR: requires high-order transfer function compared with IIR filters.
  - IIR: sensitive to finite-length arithmetic, harder to implement using fixed-point arithmetic.

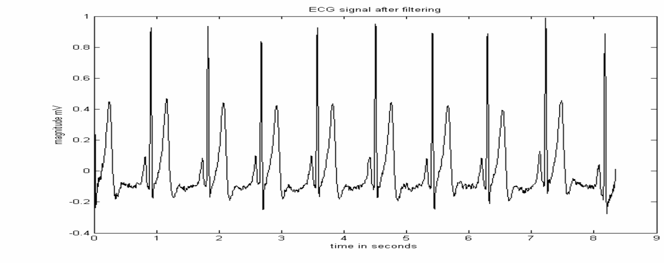
Why FIR ?

- Waveform distortion caused by phase.



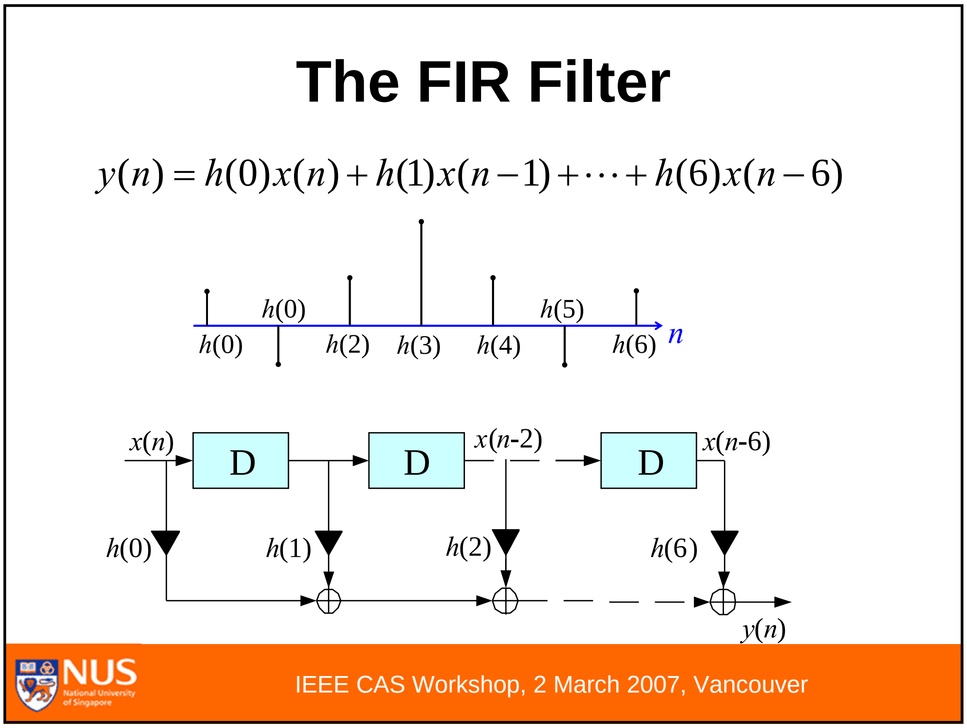


- Filtering of Electrocardiogram Signal (ECG)



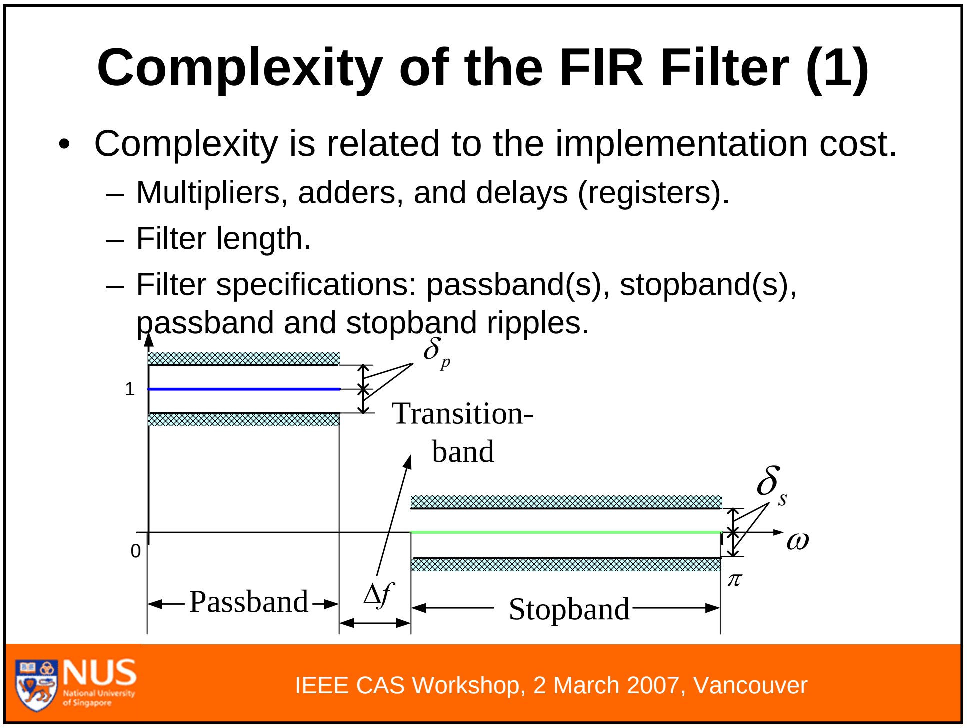
Applications

- Analog-to-digital converter.
- High quality digital audio system.
- Digital TV, HDTV.
- Wireless Communication.
- Medical instruments.
- Frequency spectrum analysis.



The FIR Filter

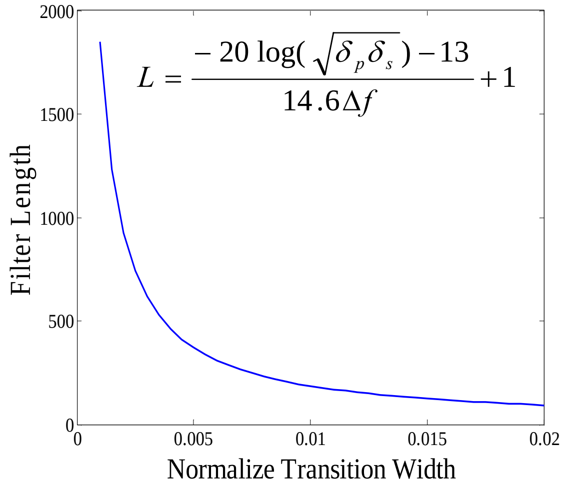
$$y(n) = h(0)x(n) + h(1)x(n-1) + \cdots + h(6)x(n-6)$$



Complexity of the FIR Filter (1)

- Complexity is related to the implementation cost.
  - Multipliers, adders, and delays (registers).
  - Filter length.
  - Filter specifications: passband(s), stopband(s), passband and stopband ripples.

Complexity of the FIR Filter (2)



Achieving High-Speed with Less Power

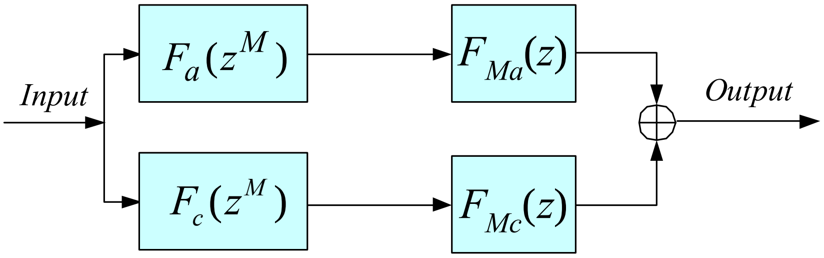
- High-speed
  - – Reduce the filter length, i.e. the number of coefficients.
  - – Reduce coefficient word-length.
  - – Remove the multipliers if possible.
- Low-power
  - – Reduce the filter length.
  - – Lower the coefficient sensitivity.
  - – Use simple multipliers.

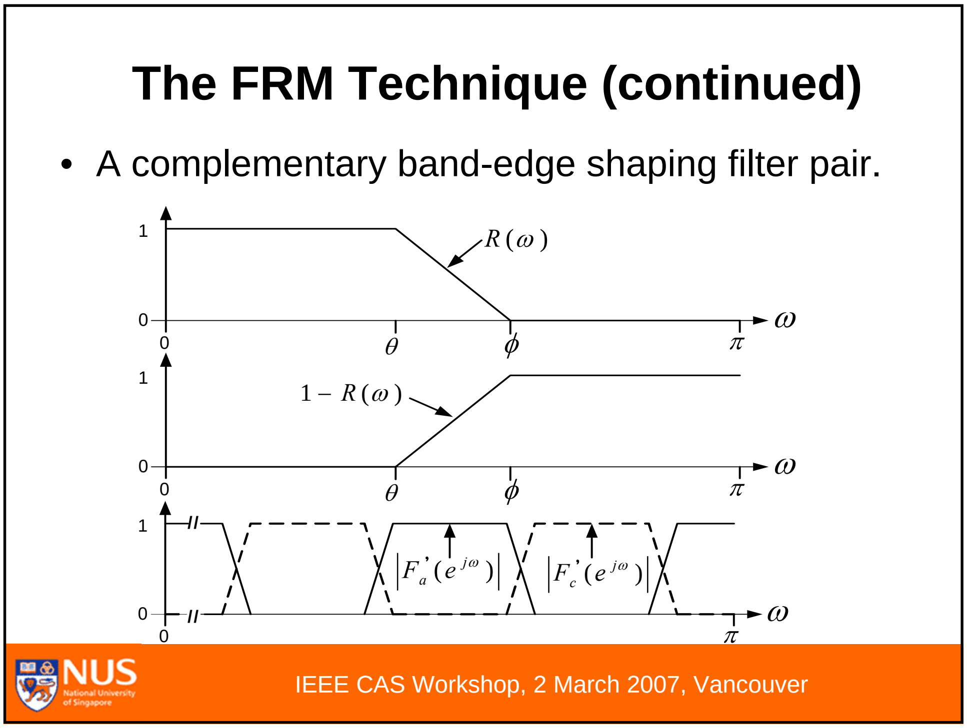
Computationally Efficient Filter Design Techniques

- Prefilter-Equalizer
  - Mainly for narrowband filters
- Interpolated Finite Impulse Response (IFIR)
  - For narrowband filters
- Frequency-Response Masking (FRM)
  - For arbitrary bandwidth narrow transition width filters

Frequency-Response Masking Technique

- It is a filter structure that realizes arbitrary bandwidth sharp FIR filter specifications.
- Basic structure of an FRM filter.





The FRM Technique (continued)

- A complementary band-edge shaping filter pair.

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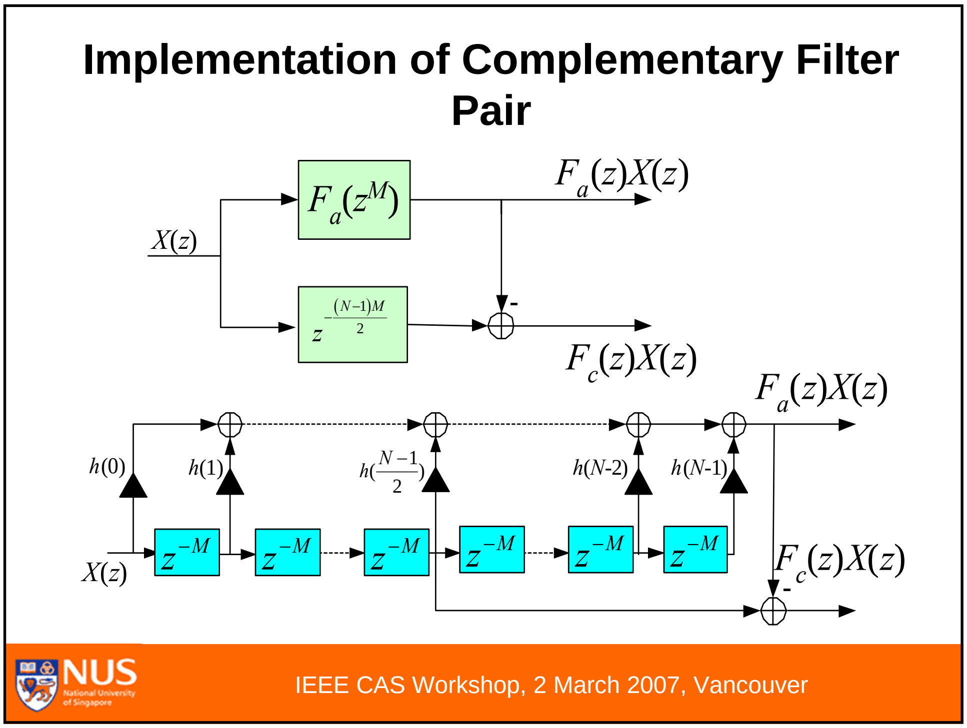

Implementation of Complementary Filter Pair

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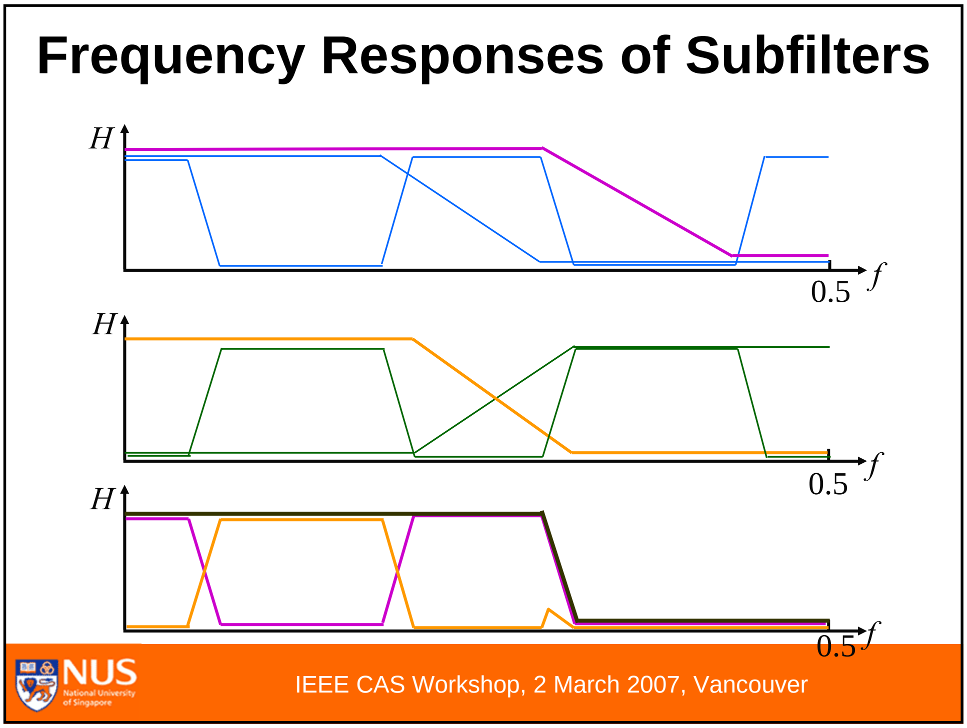

Frequency Responses of Subfilters

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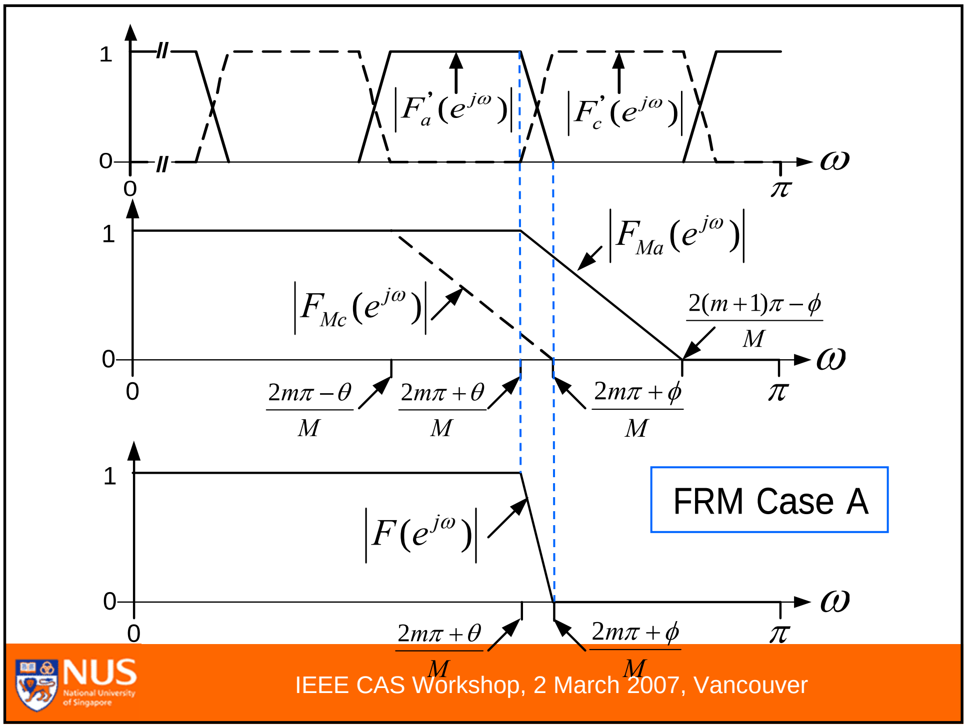

FRM Case A

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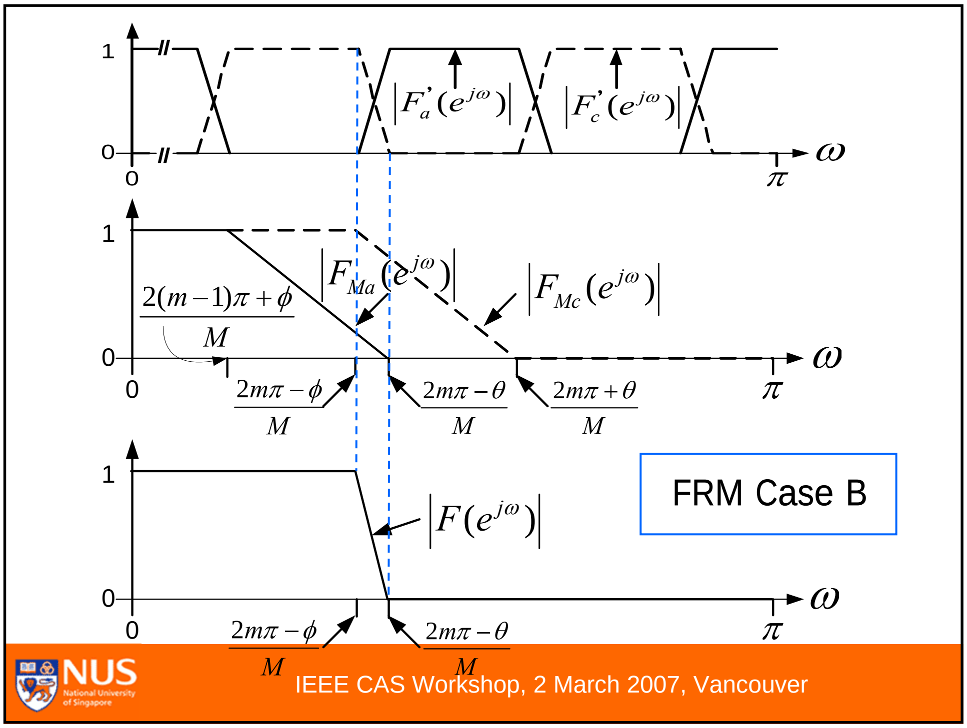

FRM Case B

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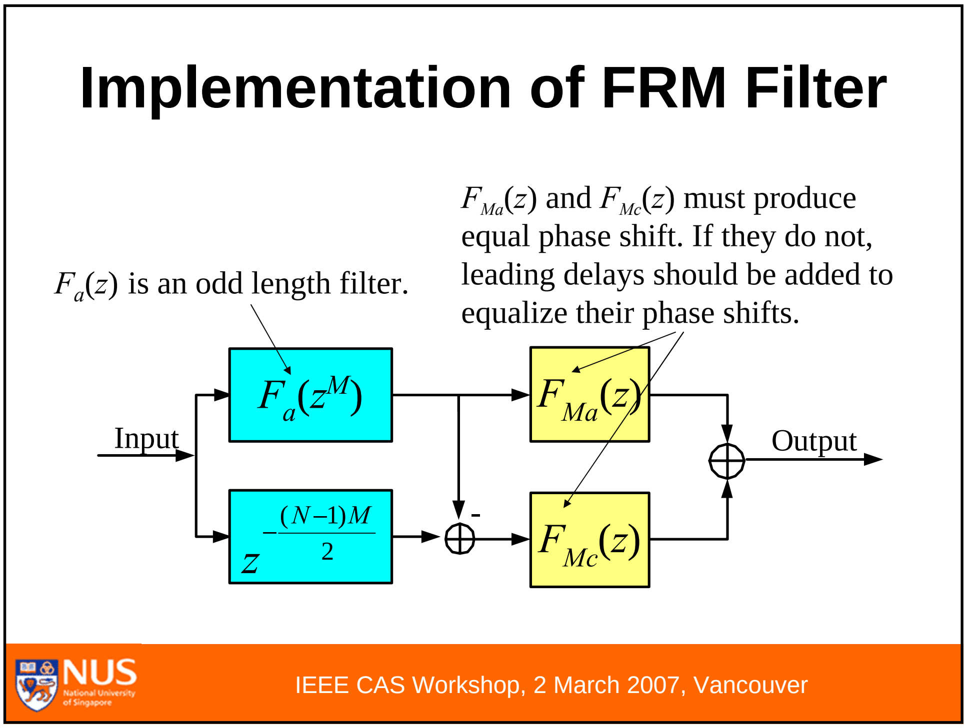
Implementation of FRM Filter



$F_a(z)$ is an odd length filter.

$F_{Ma}(z)$ and $F_{Mc}(z)$ must produce equal phase shift. If they do not, leading delays should be added to equalize their phase shifts.

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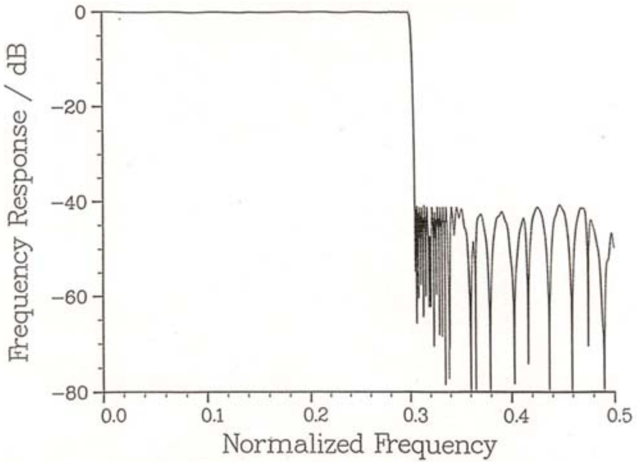
Frequency Response of an FRM Filter



Design Equations for Case A

For $F_a(z)$:

$$\omega_p = \frac{2m\pi + \theta}{M}$$

$$\omega_s = \frac{2m\pi + \phi}{M}$$

$$m = \lfloor \omega_p M / (2\pi) \rfloor$$

$$\theta = \omega_p M - 2m\pi$$

$$\phi = \omega_s M - 2m\pi$$

$$0 < \theta < \phi < \pi$$

where $\lfloor x \rfloor$ denotes the largest integer less than x; ω_p and ω_s are the passband and stopband edges of overall filter, respectively.

Design Equations for $F_{Ma}(z)$ and $F_{Mc}(z)$

- For $F_{Ma}(z)$: $\omega_{Ma,p} = \omega_p$

$$\omega_{Ma,s} = \frac{2(m+1)\pi - \phi}{M}$$

- For $F_{Mc}(z)$:

$$\omega_{Mc,p} = \frac{2m\pi - \theta}{M}$$

$$\omega_{Mc,s} = \omega_s$$

Design Equations for Case B

For $F_a(z)$:

$$\omega_p = \frac{2m\pi - \phi}{M}$$

$$\omega_s = \frac{2m\pi - \theta}{M}$$

$$m = \lceil \omega_s M/(2\pi) \rceil$$

$$\theta = 2m\pi - \omega_s M$$

$$\phi = 2m\pi - \omega_p M$$

$$0 < \theta < \phi < \pi$$

where $\lceil x \rceil$ denotes the smallest integer larger than x; ω_p and ω_s are the passband edge and stopband edge, respectively.

Design Equations for $F_{Ma}(z)$ and $F_{Mc}(z)$

- For $F_{Ma}(z)$:

$$\omega_{Ma,p} = \frac{2(m-1)\pi + \phi}{M}$$

$$\omega_{Ma,s} = \omega_s$$

- For $F_{Mc}(z)$:

$$\omega_{Mc,p} = \omega_p$$

$$\omega_{Mc,s} = \frac{2m\pi + \theta}{M}$$

Transition Width of Masking Filters

- The sum of the transition widths of two masking filters equals to 1/M.

The transition width of $F_{Ma}(z): \Delta\omega_{F_{Ma}} = \frac{2\pi - \theta - \phi}{M}$

The transition width of $F_{Mc}(z): \ \Delta\omega_{F_{Mc}} = \frac{\theta + \phi}{M}$

$$\therefore \Delta\omega_{F_{Ma}} + \Delta\omega_{F_{Mc}} = \frac{2\pi}{M}$$

The Complexity of Overall Filter

- The complexity of overall filter is the total number of multipliers needed by three subfilters,

$$L_{Total} = L_a + L_{Ma} + L_{Mc} = \frac{L_0}{M} + \frac{\phi(\delta_p,\delta_s)}{\frac{2\pi}{M} - \gamma} + \frac{\phi(\delta_p,\delta_s)}{\gamma}$$

$$= \frac{L_0}{M} + \frac{2\pi\beta L_0}{\frac{4m\pi + 2\pi - M\left(\omega_s + \omega_p\right)}{M}} + \frac{2\pi\beta L_0}{\frac{M\left(\omega_s + \omega_p\right) - 4m\pi}{M}}$$

There is no closed-form solution for the above

Complexity of the FRM Filters

- The total number of multipliers L is given by

$$L = L_a + L_{Ma} + L_{Mc} \approx (\frac{1}{M} + 4M\beta)L_o$$

- The near optimal interpolation factor can be obtained:

$$M_{opt} \approx \frac{1}{2\sqrt{\beta}}$$

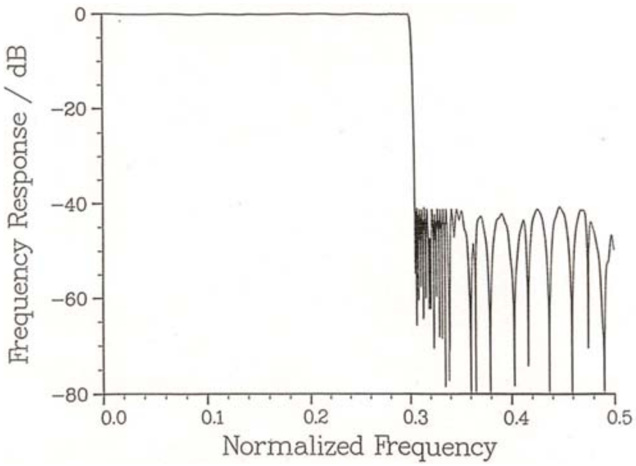
- The minimum complexity is: $L_{\min} \approx 4\sqrt{\beta}L_o$

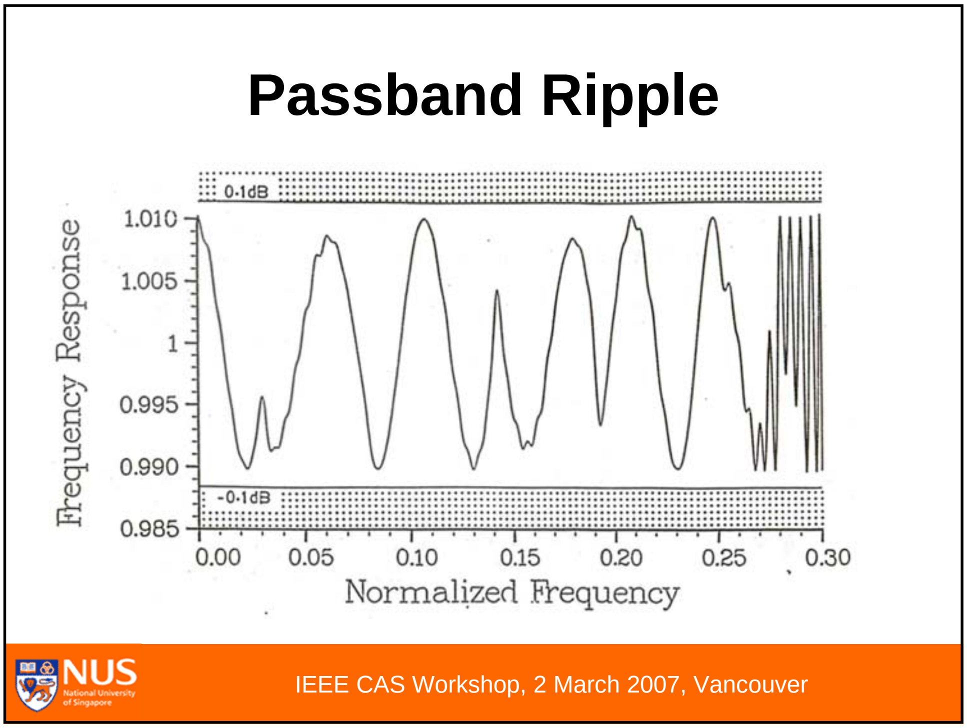
The FRM is only effective if the normalized transition bandwidth is less than 0.063.

An Example

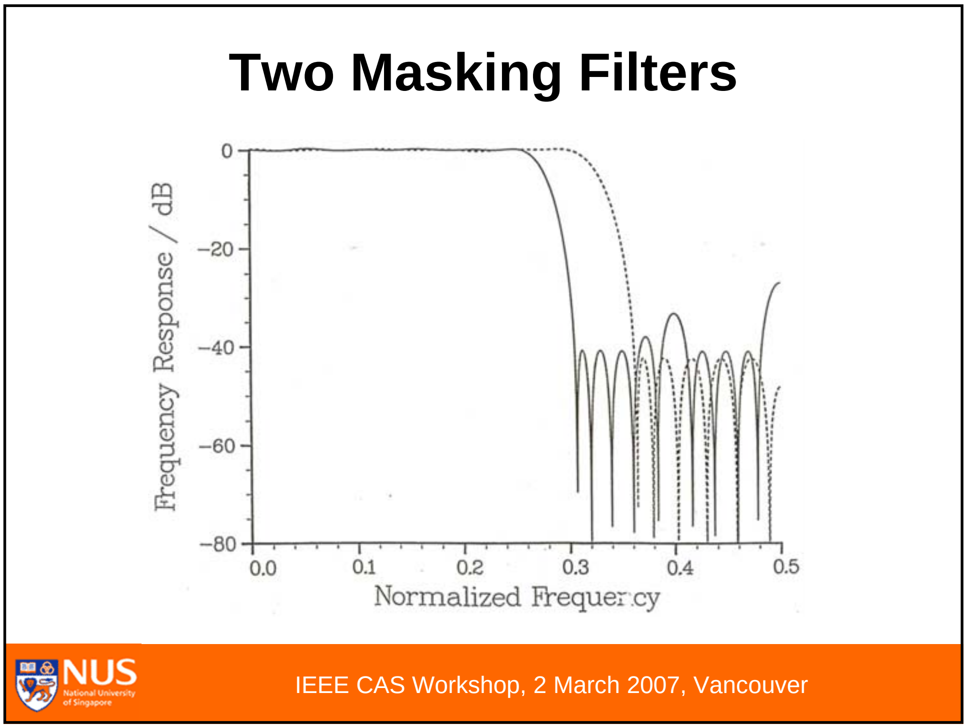
- Design an FIR lowpass filter
  Normalized passband edge: 0.3
  Normalized stopband edge: 0.305
  Maximum passband deviation: 0.01
  Minimum stopband attenuation: 40 dB
- The estimated length of the minimax design is 383, i.e. 192 multipliers.
- The lengths of filters in an FRM design are 45, 38, and 30, respectively, i.e. 57 multipliers. A 70% savings in terms of the number of multipliers compared to the minimax design.

Frequency Response of the Overall Filter





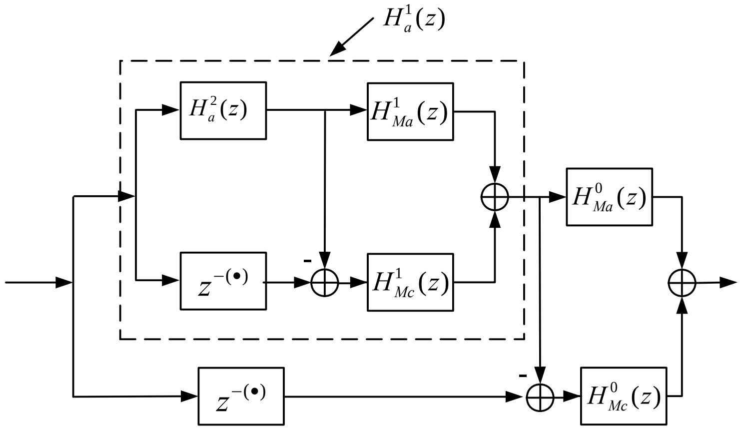
Passband Ripple



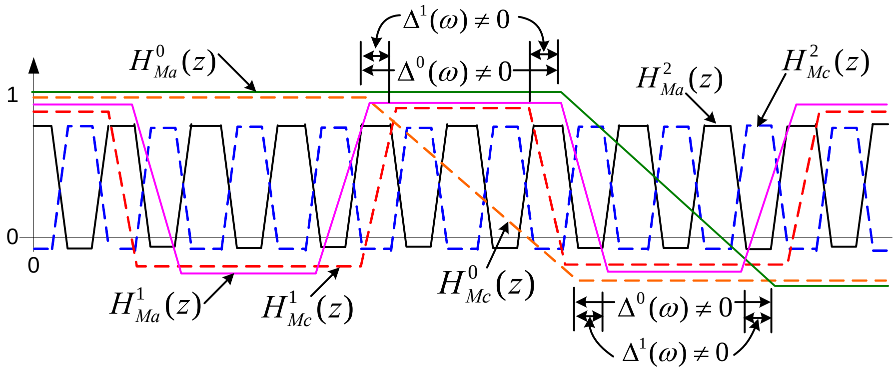
Two Masking Filters

Multi-Stage FRM

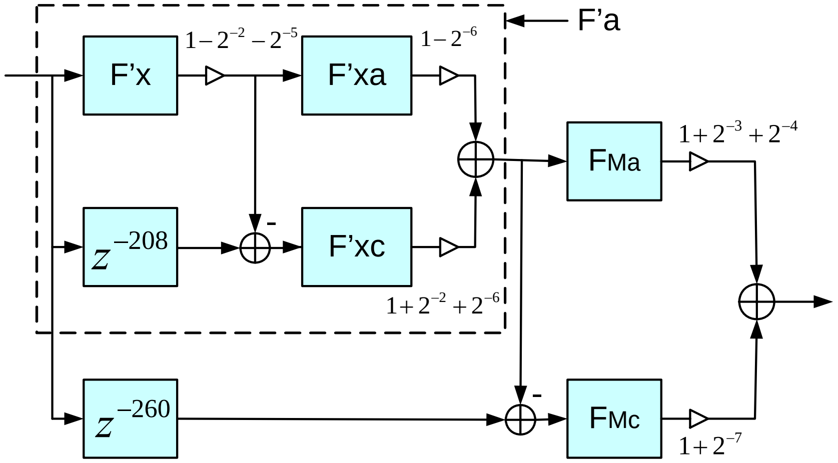
- A two-stage FRM structure



The Frequency Responses of the Various Subfilters in a Two-stage FRM



An Example of a 2-Stage Design



List of Coefficients

$h_x(0) = 2^{-8}+2^{-9} = h_x(416)$
$h_x(16) = -2^{-7}-2^{-9} = h_x(400)$
$h_x(32) = -2^{-6} = h_x(384)$
$h_x(48) = -2^{-6}+2^{-9} = h_x(368)$
$h_x(64) = 2^{-7}+2^{-8} = h_x(352)$
$h_x(80) = 2^{-5}+2^{-8} = h_x(336)$
$h_x(96) = 2^{-5}+2^{-10} = h_x(320)$
$h_x(112) = -2^{-6}-2^{-9} = h_x(304)$
$h_x(128) = -2^{-4}-2^{-6} = h_x(288)$
$h_x(144) = -2^{-4}-2^{-7} = h_x(272)$
$h_x(160) = 2^{-6}+2^{-9} = h_x(256)$
$h_x(176) = 2^{-3}+2^{-4} = h_x(240)$
$h_x(192) = 2^{-2}+2^{-3} = h_x(224)$
$h_x(208) = 2^{-1}-2^{-4}$

$h_{xa}(0) = -2^{-6} = h_{xa}(48)$
$h_{xa}(4) = -2^{-5}+2^{-8} = h_{xa}(44)$
$h_{xa}(8) = -2^{-7} = h_{xa}(40)$
$h_{xa}(12) = 2^{-5}+2^{-6} = h_{xa}(36)$
$h_{xa}(16) = 2^{-3}+2^{-6} = h_{xa}(32)$
$h_{xa}(20) = 2^{-2}-2^{-6} = h_{xa}(28)$
$h_{xa}(24) = 2^{-2}+2^{-6}$

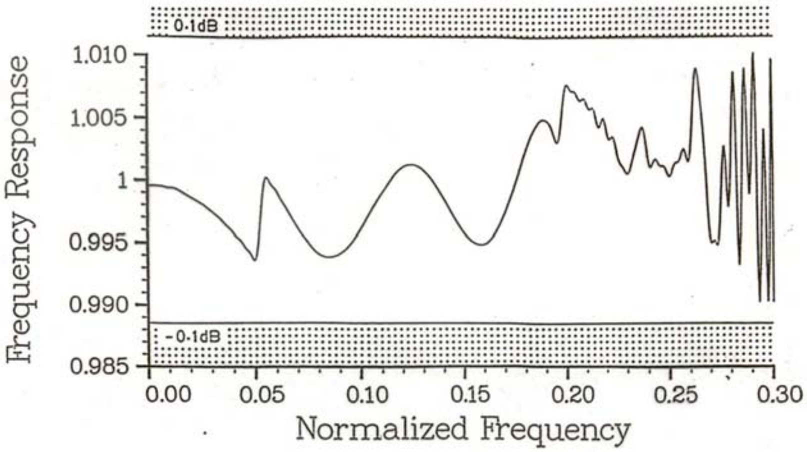
$h_{Ma}(0) = -2^{-8}-2^{-9} = h_{Ma}(14)$
$h_{Ma}(1) = 2^{-6}+2^{-9} = h_{Ma}(13)$
$h_{Ma}(2) = -2^{-6}-2^{-8} = h_{Ma}(12)$
$h_{Ma}(3) = 2^{-8}+2^{-10} = h_{Ma}(11)$
$h_{Ma}(4) = 2^{-5}+2^{-6} = h_{Ma}(10)$
$h_{Ma}(5) = -2^{-3}+2^{-9} = h_{Ma}(9)$
$h_{Ma}(6) = 2^{-3}+2^{-4} = h_{Ma}(8)$
$h_{Ma}(7) = 2^{-1}+2^{-3}$

List of Coefficients

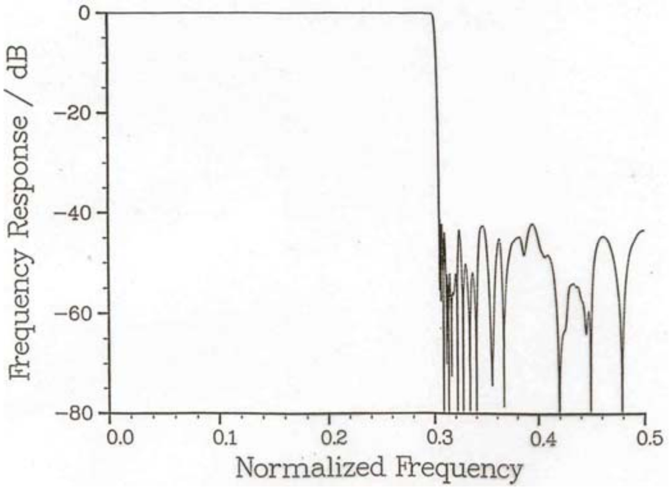
$h_{xc}(0) = 2^{-10} = h_{xc}(104)$
$h_{xc}(4) = -2^{-7}+2^{-10} = h_{xc}(100)$
$h_{xc}(8) = -2^{-8}-2^{-9} = h_{xc}(96)$
$h_{xc}(12) = 2^{-8}+2^{-9} = h_{xc}(92)$
$h_{xc}(16) = 2^{-7}+2^{-9} = h_{xc}(88)$
$h_{xc}(20) = -2^{-6}+2^{-9} = h_{xc}(84)$
$h_{xc}(24) = -2^{-6}-2^{-7} = h_{xc}(80)$
$h_{xc}(28) = 2^{-7}+2^{-8} = h_{xc}(76)$
$h_{xc}(32) = 2^{-5}+2^{-7} = h_{xc}(72)$
$h_{xc}(36) = -2^{-6}-2^{-10} = h_{xc}(68)$
$h_{xc}(40) = -2^{-4}-2^{-6} = h_{xc}(64)$
$h_{xc}(44) = 2^{-7}+2^{-8} = h_{xc}(60)$
$h_{xc}(48) = 2^{-2}-2^{-8} = h_{xc}(56)$
$h_{xc}(52) = 2^{-2}+2^{-3}$

$h_{Mc}(0) = -2^{-8} = h_{Mc}(22)$
$h_{Mc}(1) = -2^{-8} = h_{Mc}(21)$
$h_{Mc}(2) = 2^{-7}+2^{-8} = h_{Mc}(20)$
$h_{Mc}(3) = 2^{-8} = h_{Mc}(19)$
$h_{Mc}(4) = -2^{-6}-2^{-7} = h_{Mc}(18)$
$h_{Mc}(5) = -2^{-7} = h_{Mc}(17)$
$h_{Mc}(6) = 2^{-5}+2^{-6} = h_{Mc}(16)$
$h_{Mc}(7) = 2^{-7} = h_{Mc}(15)$
$h_{Mc}(8) = -2^{-4}-2^{-5} = h_{Mc}(14)$
$h_{Mc}(9) = -2^{-7}-2^{-8} = h_{Mc}(13)$
$h_{Mc}(10) = 2^{-2}+2^{-4} = h_{Mc}(12)$
$h_{Mc}(11) = 2^{-1}+2^{-8}$

The Passband Ripple



The Frequency Response of the Overall Filter



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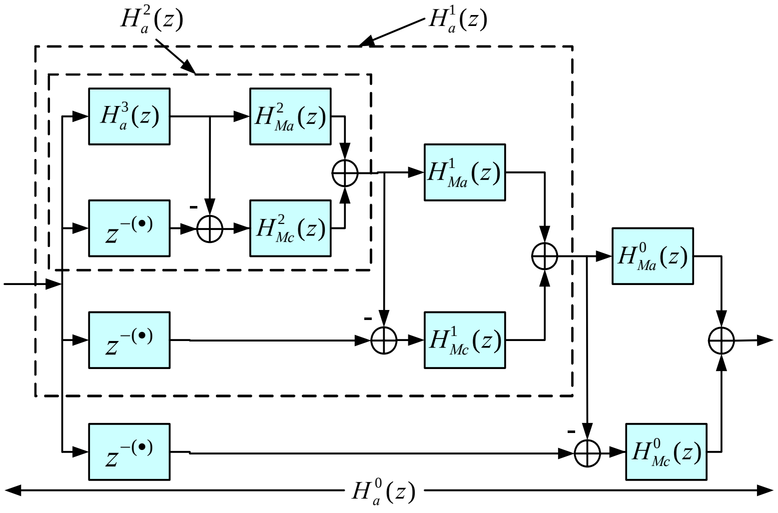
A Three-Stage Structure



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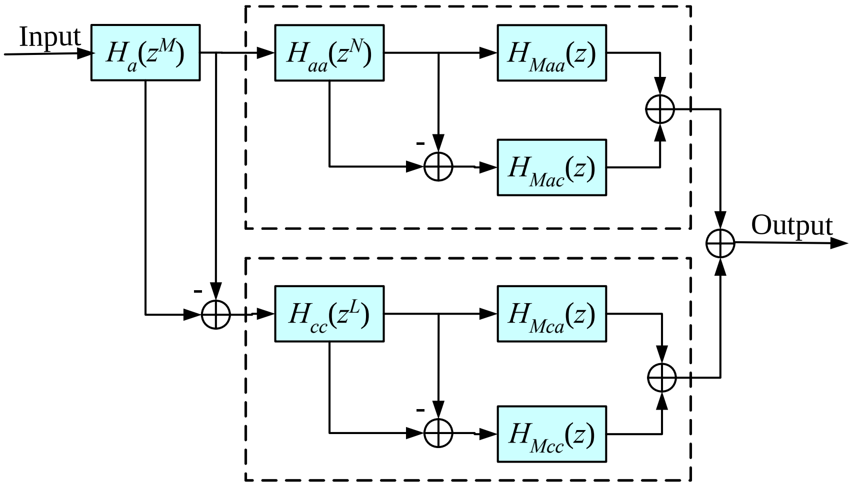
Other Multi-Stage Structure



Conclusion

- Frequency-response masking technique provides a cost efficient way for the design of high-speed low-power FIR digital filters.
- FRM significantly reduces the number of coefficients → low-power and high-speed.
- The savings in terms of number of multipliers increase with the decrease of transition bandwidth.
- FRM filters have low coefficient sensitivity and its coefficients are easy to quantize into powers-of-two terms.
- FRM filters require less number of bits → further reduction in power consumption.

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