The Design of High Speed Low Power Digital FIR Filters Based on Frequency-Response Masking Technique

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Outline

• A brief introduction to digital filters
• How to achieve high-speed with less power
• The frequency-response masking technique
• Conclusion
Digital Filters

• What is the digital filter?

• Two types of filters – Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters.

Advantages and Disadvantages

• Advantages :
  – IIR: computationally efficient.

• Disadvantages :
  – FIR: requires high-order transfer function compared with IIR filters.
  – IIR: sensitive to finite-length arithmetic, harder to implement using fixed-point arithmetic.
Why FIR?

- Waveform distortion caused by phase.
- Filtering of Electrocardiogram Signal (ECG)

Applications

- Analog-to-digital converter.
- High quality digital audio system.
- Digital TV, HDTV.
- Wireless Communication.
- Medical instruments.
- Frequency spectrum analysis.
The FIR Filter

\[ y(n) = h(0)x(n) + h(1)x(n-1) + \cdots + h(6)x(n-6) \]

Complexity of the FIR Filter (1)

- Complexity is related to the implementation cost.
- Multipliers, adders, and delays (registers).
- Filter length.
- Filter specifications: passband(s), stopband(s), passband and stopband ripples.
Achieving High-Speed with Less Power

- **High-speed**
  - Reduce the filter length, i.e. the number of coefficients.
  - Reduce coefficient word-length.
  - Remove the multipliers if possible.

- **Low-power**
  - Reduce the filter length.
  - Lower the coefficient sensitivity.
  - Use simple multipliers.
Computationally Efficient Filter Design Techniques

• Prefilter-Equalizer
  – Mainly for narrowband filters
• Interpolated Finite Impulse Response (IFIR)
  – For narrowband filters
• Frequency-Response Masking (FRM)
  – For arbitrary bandwidth narrow transition width filters

Frequency-Response Masking Technique

• It is a filter structure that realizes arbitrary bandwidth sharp FIR filter specifications.
• Basic structure of an FRM filter.

\[ F_a(z^M) \quad F_{Ma}(z) \]
\[ F_c(z^M) \quad F_{Mc}(z) \]
The FRM Technique (continued)

- A complementary band-edge shaping filter pair.

![Diagram of FRM Technique]

Implementation of Complementary Filter Pair

![Diagram of Filter Implementation]
Frequency Responses of Subfilters

IEEE CAS Workshop, 2 March 2007, Vancouver
Implementation of FRM Filter

$F_a(z)$ is an odd length filter. $F_{Ma}(z)$ and $F_{Mc}(z)$ must produce equal phase shift. If they do not, leading delays should be added to equalize their phase shifts.

$F_a(z^M)$ and $F_{Ma}(z)$ are implemented as:

$\frac{(N-1)M}{z^2}$
Frequency Response of an FRM Filter

Design Equations for Case A

For $\mathcal{F}_{\delta}(z)$:

\[
\begin{align*}
\omega_p &= \frac{2m\pi + \theta}{M} \\
\omega_s &= \frac{2m\pi + \phi}{M}
\end{align*}
\]

\[
\begin{align*}
m &= \left\lfloor \frac{\omega_p M}{2\pi} \right\rfloor \\
\theta &= \omega_p M - 2m\pi \\
\phi &= \omega_s M - 2m\pi
\end{align*}
\]

where $\lfloor x \rfloor$ denotes the largest integer less than $x$; $\omega_p$ and $\omega_s$ are the passband and stopband edges of overall filter, respectively.
Design Equations for $F_{Ma}(z)$ and $F_{Mc}(z)$

- For $F_{Ma}(z)$:
  \[
  \omega_{Ma,p} = \omega_p \\
  \omega_{Ma,s} = \frac{2(m+1)\pi - \phi}{M}
  \]

- For $F_{Mc}(z)$:
  \[
  \omega_{Mc,p} = \frac{2m\pi - \theta}{M} \\
  \omega_{Mc,s} = \omega_s
  \]

Design Equations for Case B

For $F_{d}(z)$:

\[
\begin{align*}
\omega_p &= \frac{2m\pi - \phi}{M} \\
\omega_s &= \frac{2m\pi - \theta}{M}
\end{align*}
\]

\[
\begin{align*}
m &= \left\lfloor \frac{\omega_s M}{(2\pi)} \right\rfloor \\
\theta &= 2m\pi - \omega_s M \\
\phi &= 2m\pi - \omega_p M \\
0 &< \theta < \phi < \pi
\end{align*}
\]

where $\lfloor x \rfloor$ denotes the smallest integer larger than $x$; $\omega_p$ and $\omega_s$ are the passband edge and stopband edge, respectively.
Design Equations for $F_{Ma}(z)$ and $F_{Mc}(z)$

- For $F_{Ma}(z)$:
  \[ \omega_{Ma,p} = \frac{2(m-1)\pi + \phi}{M} \]
  \[ \omega_{Ma,s} = \omega_s \]

- For $F_{Mc}(z)$:
  \[ \omega_{Mc,p} = \omega_p \]
  \[ \omega_{Mc,s} = \frac{2m\pi + \theta}{M} \]

Transition Width of Masking Filters

- The sum of the transition widths of two masking filters equals to $1/M$.

  The transition width of $F_{Ma}(z)$: $\Delta \omega_{F_{Ma}} = \frac{2\pi - \theta - \phi}{M}$

  The transition width of $F_{Mc}(z)$: $\Delta \omega_{F_{Mc}} = \frac{\theta + \phi}{M}$

  \[ \therefore \Delta \omega_{F_{Ma}} + \Delta \omega_{F_{Mc}} = \frac{2\pi}{M} \]
The Complexity of Overall Filter

- The complexity of overall filter is the total number of multipliers needed by three subfilters,

\[
L_{\text{Total}} = L_a + L_{Ma} + L_{Mc} = \frac{L_0}{M} + \frac{\phi(\delta_p, \delta_s)}{2\pi M - \gamma} + \frac{\phi(\delta_p, \delta_s)}{\gamma}
\]

- There is no closed-form solution for the above

Complexity of the FRM Filters

- The total number of multipliers \( L \) is given by

\[
L = L_a + L_{Ma} + L_{Mc} \approx \left( \frac{1}{M} + 4M \beta \right)L_0
\]

- The near optimal interpolation factor can be obtained:

\[
M_{\text{opt}} \approx \frac{1}{2\sqrt{\beta}}
\]

- The minimum complexity is:

\[
L_{\min} \approx 4\sqrt{\beta}L_0
\]

The FRM is only effective if the normalized transition bandwidth is less than 0.063.
An Example

- Design an FIR lowpass filter
  Normalized passband edge: 0.3
  Normalized stopband edge: 0.305
  Maximum passband deviation: 0.01
  Minimum stopband attenuation: 40 dB
- The estimated length of the minimax design is 383, i.e. 192 multipliers.
- The lengths of filters in an FRM design are 45, 38, and 30, respectively, i.e. 57 multipliers. A 70% savings in terms of the number of multipliers compared to the minimax design.

Frequency Response of the Overall Filter
Passband Ripple

Two Masking Filters
Multi-Stage FRM

- A two-stage FRM structure

The Frequency Responses of the Various Subfilters in a Two-stage FRM
An Example of a 2-Stage Design

List of Coefficients

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### The Passband Ripple

![The Passband Ripple Graph](image-url)
The Frequency Response of the Overall Filter

A Three-Stage Structure

\[
\begin{align*}
H_1^2(z) & \quad H_1^3(z) \\
H_1^1(z) & \quad H_2^1(z) \\
z^{-1} & \quad H_2^2(z) \\
z^{-1} & \quad H_3^2(z) \\
z^{-1} & \quad H_3^3(z) \\
H_0^0(z) & \quad H_0^3(z) \\
H_0^1(z) & \quad H_0^2(z)
\end{align*}
\]
Other Multi-Stage Structure

Input $H_a(z^M)$ $H_{na}(z^N)$ $H_{Mab}(z)$ $H_{Mac}(z)$ $H_{Mc}(z)$ $H_{Mcc}(z)$ $H_{cc}(z^L)$ $H_{Mca}(z)$ $H_{M}(z)$

Conclusion

• Frequency-response masking technique provides a cost efficient way for the design of high-speed low-power FIR digital filters.
• FRM significantly reduces the number of coefficients $\rightarrow$ low-power and high-speed.
• The savings in terms of number of multipliers increase with the decrease of transition bandwidth.
• FRM filters have low coefficient sensitivity and its coefficients are easy to quantize into powers-of-two terms.
• FRM filters require less number of bits $\rightarrow$ further reduction in power consumption.
References


